Load-transfer curves: from field data to engineering values

Rob van der Salm [i)]

i) Technical director, CSGEO LLP, Atyrau, Kazakhstan.

ABSTRACT

The load-transfer method is an efficient and practical tool for the analysis of single piles. Load-settlement behaviour analyses of piles are getting more and more important, compared to solely bearing capacity analyses. This tendency is reflected in the European standard for geotechnics Euro-code 7. The axial load-transfer or 't-z' method describes the pile shaft and pile tip local resistance mobilization as a function of the pile axial displacement at the corresponding depth. It represents a very straightforward and practical method for isolated piles.

The load-transfer curves can be defined in different ways from different soil parameters and they are based on theory and on experience. The estimation of the ultimate value of the friction and the tip resistance depends mostly on regional ground properties and on local practice. The load-transfer method is thus an intermediate tool between fundamental soil mechanics and empirical approaches.

Results from a static pile load test can be very erratic and seemingly uncorrelated. With a load-transfer curve, a best fit through the data points can be created and an estimate of required geotechnical parameters can be obtained. A simple spreadsheet can be used to assess the test results.

This paper describes how to create a model for basic geotechnical and structural parameters that will match the acquired data from a pile load test.

Keywords: Pile testing, load test result analyses, geotechnical engineering, symposium , load-settlement behaviour analyses, load-transfer curves

1 INTRODUCTION

The load-transfer method is an efficient and practical tool for the analysis of single piles. Load-settlement behaviour analyses of piles are getting more and more important compared to solely bearing capacity analyses. This tendency is reflected in the European standard for geotechnics Euro-code 7. The axial load-transfer or 't-z' method describes the pile shaft and pile tip local resistance mobilization as a function of the pile axial displacement at the corresponding depth. It represents a very straightforward and practical method for isolated piles.

The load-transfer curves can be defined in different ways from different soil parameters and they are based on theory and on experience. The estimation of the ultimate value of the friction and of the tip resistance depends mostly on regional ground properties and on local experience. The load-transfer method is thus an intermediate tool between fundamental soil mechanics like Plaxis and empirical approaches like Chin-Kondner.

Results from a static pile load test can be very erratic and seemingly uncorrelated. With a load-transfer curve, a best fit through the data points can be created and an estimate of required geotechnical parameters can be obtained. It should be noted that it is very important that special attention is taken during the pile load test. To acquire the best results, the author recommends to specially observe the following points during the execution of the test:

A) The maximum applied load should preferably exceed the ultimate skin friction and/or be close to the ultimate capacity of the pile;
B) Enough load steps should be taken. Minimum one step for each geotechnical parameter to solve;
C) Sufficient time for each load step to meet the given creep threshold, so an assessment can be made of the settlement asymptote at the load step;
D) The applied load should be maintained with high accuracy (preferably automatic) to provide stable results.

The advantage of using the load-transfer model is that from only top-down test information, multiple geotechnical parameters can be derived. Once these parameters are set, the engineer or consultant can easily investigate the effects of changing the diameter, change of elastic modulus of the pile or length of the pile to optimize the design.

This paper describes a practical route to extract geotechnical parameters from load test results with different load-transfer models. The general model applies a curve fit that minimizes the error between $\Delta_{t\_Observed}$ and $\Delta_{t\_modelled}$ using the 'build-in' spreadsheet solver. The model can easily be setup in your favourite spreadsheet program without using macro's, scripting or software development tools.

Although some comparison between the different solutions is given, the author does not advocate which model will give the best results under all circumstances.

2 DEFINITIONS

In this paper assumes that all information is in SI-units. With some minor changes, the same method and formulae can also be used with imperial or local units.

Table 1 General parameters

Parameter	Description	Units
F_t	Load applied at top of pile	kN
Δ_t	Displacement top of pile	m
F_{ut}	Ultimate load capacity of pile	kN
F_s	Load consumed by skin friction	kN
Δ_s	Displacement at start level of friction	m
F_{us}	Ultimate load capacity of shaft	kN
A_s	Average shaft cross section for friction	m^2
D_s	Equivalent diameter of shaft	m
F_b	Load consumed at the base	kN
Δ_b	Displacement at the base	m
F_{ub}	Ultimate load capacity of base	kN
A_b	Cross section at base	m^2
D_b	Equivalent diameter of base	m
E_c	Elastic modulus pile	kN/m^2
L_f	Friction length	m
L_0	Friction free length (stick-up)	m
K	Friction transfer factor	-
Δ_{lf}	Elastic shortening of pile over friction length	m
Δ_{l0}	Elastic shortening of pile over non friction length (stick-up)	m

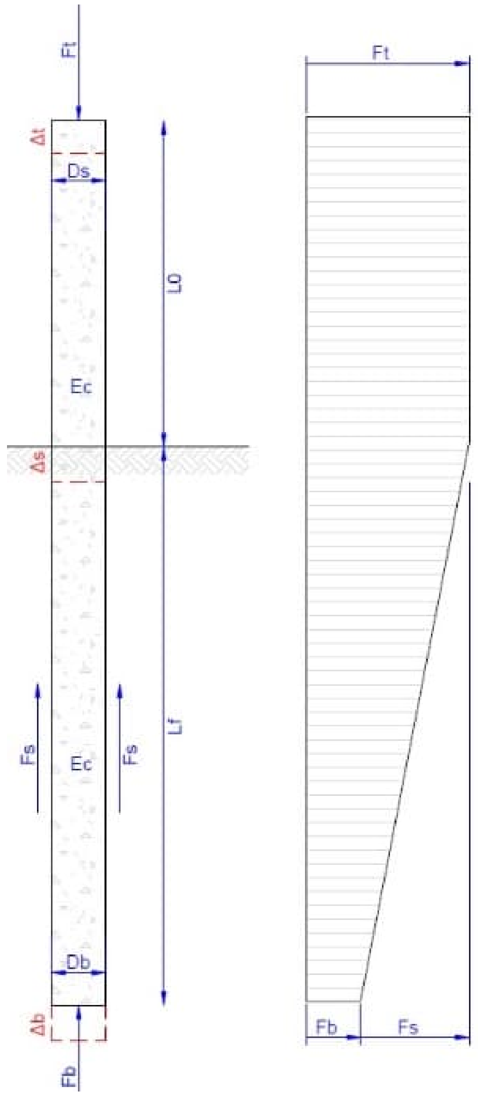

Figure 1 Pile load parameters

Table 2 Solution specific parameters.

Parameter	Description	Units
E_{ms}	Pressure meter test results at level of the shaft	MPa
E_{mb}	Pressure meter test result at the level of the base	MPa
α	For fine grained soils, α= 2.0, for coarse grained soils α= 0.8	-
β	For fine grained soils β=11 and for coarse grained soils β=4.8	-
M_s	Flexibility factor of pile shaft/soil interface	-
M_b	Flexibility factor of pile base/soil interface	-

3 MULTI LAYER SOLUTION AND COMBINING MODELS

Although this paper only concentrates on a single layer model, a multilayer model is also possible, but makes the calculation sheet more complex. In case of a multilayer model, the F_b and Δ_b from the upper section, are to be used as input (F_s ,Δ_s) for the lower section.

The modelling approach is flexible and makes it possible to combine different models to fit the soil-pile response. For example: for the skin friction (F_s,Δ_s) an hyperbolic model can be selected to be combined with the Frank and Zhao for the pile base response (F_b,Δ_b).

4 APPLYING LOAD-TRANSFER CURVES FOR BI-DIRECTIONAL TESTS

For the high capacity piles, the bi-directional method is a great method to retrieve the geotechnical parameters. The results of a single level bi-directional test can easily be used to construct a top-load displacement curve. The top section, with upward displacement, can be modelled as a pile with no end bearing which will give the skin friction over the top section of the pile. The lower section below the jacks, can be modelled as a normal pile. The total load-displacement curve can be constructed from F_{us_Total} = $F_{us\_Upwards}$ + $F_{us\_Downwards}$ and F_{ub_Total} = F_{ub}.

5 ELASTIC SHORTENING OF THE PILE

In the modelling, an unknown parameter is the elastic shortening of the pile over the friction length (Δ_{lf}). This elastic behaviour is related to F_s and F_b. The general equation for elastic shortening of material is described by Hooke's Law:

$$\Delta_l = \frac{FL}{EA} \tag{1}$$

if the pile has no friction and only depends on end bearing:

$$\Delta_{lf} = \frac{F_t L_f}{E_c A_s} \tag{2}$$

If the pile has no end bearing but only skin friction, the equation will become:

$$\Delta_{lf} = \frac{1}{2}\frac{F_t L_f}{E_c A_s} \tag{3}$$

More generalized:

$$\Delta_{lf} = K\frac{F_t L_f}{E_c A_s} \tag{4}$$

If the skin friction is uniform distributed over the full length of the pile, K may be approximated by:

$$K = \frac{F_t + F_b}{2F_t} \tag{5}$$

The elastic shortening of the pile over the friction length will be:

$$\Delta_{lf} = \left(\frac{F_t + F_b}{2F_t}\right)\frac{F_t L_f}{E_c A_s} \quad (6)$$

If we factor the 'constant values' as x_5

$$x_5 = \frac{L_f}{2E_c A_s} \quad (7)$$

$$\Delta_{lf} = x_5(F_t + F_b) \quad (8)$$

To allow for the set-up of the pile above ground level, the additional elastic shortening of the pile over the no friction zone will be:

$$\Delta_{l0} = \frac{F_t L_o}{E_c A_s} \quad (9)$$

If we factor the 'constant values' as x_6

$$x_6 = \frac{L_o}{E_c A_s} \quad (10)$$

$$\Delta_{l0} = x_6(F_t) \quad (11)$$

The total displacement at the top of the pile can be written as:

$$\Delta_t = \Delta_b + \Delta_{lf} + \Delta_{l0} \quad (12)$$

6 MODELLING LINEAR CURVES

In this chapter the base formula used are where the shaft friction and base resistance is separated and described by a linear function. This is one of the most basic solutions. The load – displacement response looks like graph below:

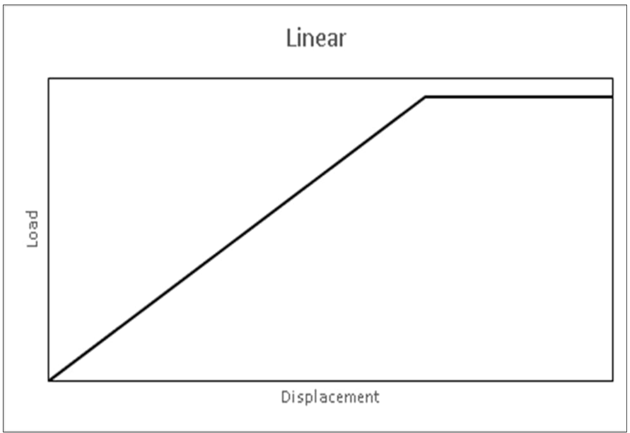

Figure 2 Linear approximation load-displacement

The generic formulae for a linear pile assessment at the base and along the skin looks like as follows:

$$F = x\Delta \quad (13)$$

Where:

$$F_t = F_s + F_b \quad (14)$$

And

$$\Delta_b = \Delta_s - \Delta_{lf} \quad (15)$$

After combining equations above:

$$x_1 F_b + x_1 x_2 \Delta_{lf} = x_s F_s \quad (16)$$

The equation can be organized as:

$$F_s = \frac{(1 + 2x_2 x_5)x_1 F_t}{(x_1 + x_2 + x_1 x_2 x_5)} \quad (17)$$

Table 3 Linear parameters Verbrugge.

	x_1	x_2
Verbrugge (1981)	$0.22 L_f \pi E_s ab$	$\frac{\pi D_b E_b ab}{4 * 0.32}$

a = 1 for bored piles and 3 for driven piles

b = 1 for normal consolidated soils and 2 for over consolidated soils

E = 3600 + 2.2 q_c

a, b, and E can also be regarded as a variable and solved from the field test data.

Table 4 Linear parameters Randolph.

	x_1	x_2
Randolph, Wroth (1978)	$\frac{\pi L_f G}{2ln\left(\frac{2R_m}{D_s}\right)}$	$\frac{2D_b G}{(1-\upsilon)}$

7 MODELLING TRI-LINEAR CURVES

In this chapter the base formula used are where the shaft friction and base resistance is separated and described by a tri-linear function like used by Frank and Zhao. The functions used are not continuous and need some additional conditions in the model. For Frank and Zhao, the discontinuity takes place at 0.5 F_u where the slope of the relation between load and displacement changes with a factor 5. The presented method in this paper is also suitable to asses this point of the discontinuity and the following slope as variable to match the test results. In the example below, the point of discontinuity and the change of slope are assumed constant as proposed by Frank and Zhao. See schematic load-displacement response below:

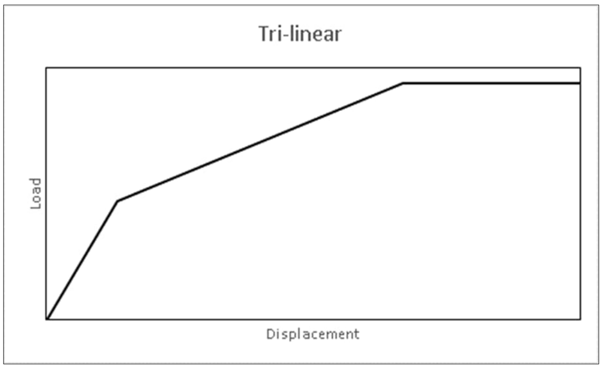

Figure 3 Tri-linear approximation load-displacement

The generic formulae for a linear pile assessment looks like as follows:

$$F_s < \frac{1}{2}F_{us}$$
$$\Delta_s = \frac{F_s}{x_1} \tag{18}$$
$$F_s \geq \frac{1}{2}F_{us}$$
$$\Delta_s = \frac{5F_s - 2F_{us}}{x_1}$$

$$F_b < \frac{1}{2}F_{ub}$$
$$\Delta_b = \frac{F_b}{x_2} \tag{19}$$
$$F_b \geq \frac{1}{2}F_{ub}$$
$$\Delta_b = \frac{5F_b - 2F_{ub}}{x_2}$$

After combining (14), (15), (18) and (19) there will be four different states that can be described by different formulae:

$$F_s < \frac{1}{2}F_{us} \wedge F_b < \frac{1}{2}F_{ub}$$
$$F_s = \frac{(x_1 + 2x_1x_2x_5)F_t}{x_1 + x_2 + x_1x_2x_5} \tag{20}$$

$$F_s \geq \frac{1}{2}F_{us} \wedge F_b < \frac{1}{2}F_{ub}$$
$$F_s = \frac{(x_1 + 2x_1x_2x_5)F_t + 2x_2F_{us}}{x_1 + 5x_2 + x_1x_2x_3} \tag{21}$$

$$F_s < \frac{1}{2}F_{us} \wedge F_b \geq \frac{1}{2}F_{ub}$$
$$F_s = \frac{(5x_1 + 2x_1x_2x_5)F_t - 2x_1F_{ub}}{5x_1 + x_2 + x_1x_2x_5} \tag{22}$$

$$F_s \geq \frac{1}{2}F_{us} \wedge F_b \geq \frac{1}{2}F_{ub}$$
$$F_s = \frac{(5x_1 + 2x_1x_2x_5)F_t + 2x_2F_{us} - 2x_1F_{ub}}{5x_1 + 5x_2 + x_1x_2x_5} \tag{23}$$

Table 5 Tri-linear parameters Frank Zhao.

	x1	x2
Frank and Zhao (1982, 1985)	$\tau_s \pi D_s L_f$	$\tau_b \frac{1}{4}\pi D_b^2$
	$\tau_s = \frac{\alpha E_{ms}}{D_s}$	$\tau_b = \frac{\beta E_{mb}}{D_b}$

Fine grained soils, α= 2.0, β=11
Coarse grained soils α= 0.8, β=4.8

The parameters α, β, E_{ms}, E_{mb} are considered known constants but can also be solved with build-in spreadsheet functions to create a site specific model.

8 MODELLING HYPERBOLIC CURVES

In this chapter the base formula used are where the shaft friction and base resistance is separated and described by an hyperbolic function.

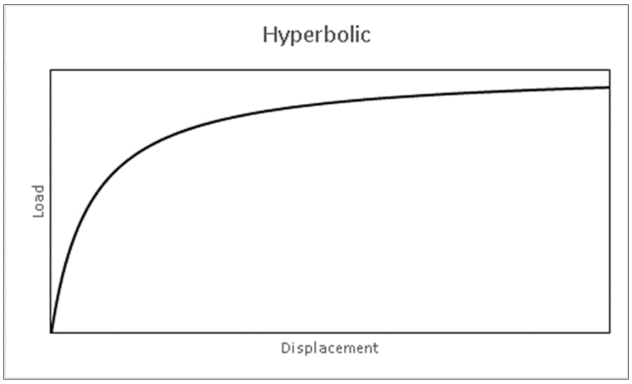

Figure 4 Hyperbolic approximation load-displacement

The generic formulae for an hyperbolic pile assessment looks like as follows:

$$F_s = \frac{x_1\Delta_s}{x_2 + \Delta_s} \tag{24}$$

$$F_b = \frac{x_3\Delta_b}{x_4 + \Delta_b} \tag{25}$$

After combining (25) and (26)

$$-x_2x_3F_s + (x_2F_s + x_1x_4 - x_4F_s)F_b + (x_1x_3 - x_1F_b - x_3F_s + F_sF_b)\Delta_{lf} = 0 \tag{26}$$

The elastic shortening can be substituted from equation (8) and (11):

$$(x_2F_s + x_1x_4 - x_4F_s + x_1x_3x_5 - x_3x_5F_s)F_b + \tag{27}$$

$$(x_5F_s - x_1x_5)F_b^2$$
$$(x_1x_3x_5 - x_1x_5F_b - x_3x_5F_s + x_5F_sF_b)F_t$$
$$-x_2x_3F_s = 0$$

Substituting equation (14) in (28) and rewrite to F_s:

$$\begin{aligned} &1F_s^3 + \\ &\frac{1}{x_5}(x_4 - x_2 - x_1x_5 + x_3x_5 - 3x_5F_t)F_s^2 + \\ &\frac{1}{x_5}(x_2F_t - x_2x_3 - x_1x_4 - x_4F_t - x_1x_3x_5 \\ &\quad + 3x_1x_5F_t - 2x_3x_5F_t \\ &\quad + 2x_5F_t^2)F_s + \\ &\frac{1}{x_5}(x_1x_4 + 2x_1x_3x_5F_t - 2x_1x_5F_t^2) = 0 \end{aligned} \tag{28}$$

For this third order polynomic equation an exact solution can be found.

There are several papers on the parameters that can be used for describing the load-settlement curve. These parameters depend on the local situation and the tests performed. Below a short selection of suggested model parameters for round piles.

Table 6 Hyperbolic parameters.

	x_1	x_2	x_3	x_4
Hirayama (1990)	F_{us}	M_sD_s $M_s = 0.0025$	F_{ub}	M_bD_b $M_b = 0.25$
Flemming (1992)	F_{us}	M_sD_s	F_{ub}	$\frac{0.6F_{ub}}{D_bE_b}$
C. Bohn; A. Lopes dos Santos; and R. Frank (2016)	F_{us}	M_sD_s $M_s = 0.0038$	F_{ub}	M_bD_b $M_b = 0.01$
Hyperbolic Non fixed parameters	F_{us}	M_sD_s	F_{ub}	M_bD_b

In some cases the flexibility factor parameters like M_s, M_b are considered constant. If there are enough load steps in the pile load test, these parameters can be regarded variable and spreadsheet functions can be used to solve variables and create a site specific model.

9 MODELLING COMBINED CURVES

The chapters above show how to model the linear, tri-linear and hyperbolic functions to assess the pile test field results. The geotechnical engineer might reason that the base follows a different model than the shaft or the national standard dictates to use a certain model. In that case it is possible to combine different approaches in one solution. The example below shows an hyperbolical function for the shaft and a tri-linear function for the base.

The generic formulae for a hyperbolic pile shaft assessment looks like as follows:

$$F_s = \frac{x_1\Delta_s}{x_2 + \Delta_s} \tag{29}$$

And the tri-linear assessment of the base:

$$\begin{aligned} &F_b < \frac{1}{2}F_{ub} \\ &\Delta_b = \frac{F_b}{x_3} \\ &F_b \geq \frac{1}{2}F_{ub} \\ &\Delta_b = \frac{5F_b - 2F_{ub}}{x_3} \end{aligned} \tag{30}$$

After combining (14), (15), (30) and (31) there will be two different states that can be described by different formulae:

$$\begin{aligned} &F_b < \frac{1}{2}F_{ub} \\ &\frac{x_2F_s}{x_1 - F_s} = \frac{F_b}{x_3} + \Delta l_f \end{aligned} \tag{31}$$

And:

$$\begin{aligned} &F_b \geq \frac{1}{2}F_{ub} \\ &\frac{x_2F_s}{x_1 - F_s} = \frac{5F_b - 2F_{ub}}{x_3} + \Delta l_f \end{aligned} \tag{32}$$

For both states, the formulae can be rewritten in a second order polynomic equation:

$$\begin{aligned} &F_b < \frac{1}{2}F_{ub} \\ &a = 1 + x_3x_5 \\ &b = -x_2x_3 - x_1 - F_t - 2x_3x_5F_t - x_1x_3x_5 \\ &c = x_1F_t + 2x_1x_3x_5F_t \end{aligned} \tag{33}$$

$$\begin{aligned} &F_b \geq \frac{1}{2}F_{ub} \\ &a = 5 + x_3x_5 \\ &b = -5(x_1 + F_t) + 2(F_{ub} - x_3x_5F_t) \\ &\quad -x_1x_3x_5 - x_2x_3 \\ &c = 5x_1F_t + 2x_1(x_3x_5F_t - F_{ub}) \end{aligned} \tag{34}$$

10 GENERAL CONDITIONS FOR ALL SOLUTIONS

When building the model, there are conditions that are evident, but should be considered:

$$F_s \leqslant F_t \wedge F_s \leqslant F_{us} \tag{35}$$

$$F_b \leqslant F_t \wedge F_b \leqslant F_{ub} \tag{36}$$

$$F_{us} + F_{ub} \geqslant F_t \tag{37}$$

$$F_t = 0 \Rightarrow \Delta_s = 0 \wedge \Delta_b = 0 \tag{38}$$

11 BUILDING THE SPREADSHEET MODEL

With the theoretical models for linear, tri-linear and hyperbolic, a suitable spreadsheet model can be created. The table below shows an generic solution to solve the hyperbolic approach to obtain the engineering values of the field test.

One of the most challenging models presented in this paper to solve in a spreadsheet, is the third order polynomic equation. To solve this some additional steps are introduced (Column C to I)

Table 7 Spreadsheet setup for hyperbolic models.

Column	Formula	Remarks
A	F_t	Top load
B	Δ_t	Observed top displacement
C	$\frac{1}{x_5}(x_4 - x_2 - x_1x_5 + x_3x_5 - 3x_5F_t)$	a
D	$\frac{1}{x_5}(x_2F_t - x_2x_3 - x_1x_4 - x_4F_t - x_1x_3x_5 + 3x_1x_5F_t - 2x_3x_5F_t + 2x_5F_t^2)$	b
E	$\frac{1}{x_5}(x_1x_4 + 2x_1x_3x_5F_t - 2x_1x_5F_t^2)$	c
F	$\frac{a^2 - 3b}{9}$	Q
G	$\frac{2a^3 - 9ab + 27c}{54}$	R
H	$ACOS\left(\frac{R}{\left(\sqrt{(Q^3)}\right)}\right)$	Θ (in radians)
I	$-\left(2\sqrt{(Q)}cos\left(\frac{\theta - 2\pi}{3}\right)\right) - \frac{a}{3}$	M (There are three solutions. Only one is shown here)
J	$\left(-R + \sqrt{(M)}\right)^{\left(\frac{1}{3}\right)} + \left(-R - \sqrt{(M)}\right)^{\left(\frac{1}{3}\right)} - \frac{a}{3}$	F_s
K	$F_t - F_s$	F_b
L	$\frac{x_2F_s}{x_1 - F_s}$	Δ_s
M	$\frac{x_4F_b}{x_3 - F_b}$	Δ_b
N	$\left(\frac{F_t + F_b}{2F_t}\right)\frac{F_tL_f}{E_cA_s}$	Δ_{lf}
O	$\Delta_s + F_tx_6$	$\Delta_{t\_Calculated}$
P	$\left(\Delta_{t_{observed}} - \Delta_{t_{calculated}}\right)^2$	Error squared

To optimize the solution for the given dataset, the sum of squared errors needs to be minimized. For this, the build-in solver function of the spreadsheet can be utilized.

12 EXAMPLE DATA SET

The following dataset is obtained from an actual pile test. The test was executed between 2015 and 2016 as part of a large pile testing campaign with over 100 tests. The location cannot be disclosed. The data was collected every 10 seconds. The raw data is shown below. The data points used for the analyse are indicated in red.

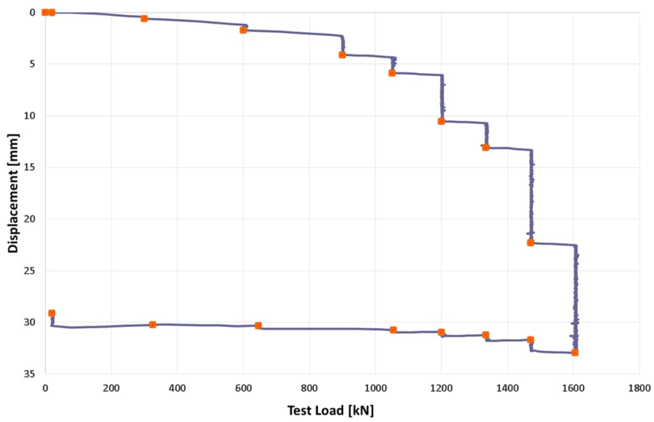

Figure 5 Pile test data set

Table 8 Pile test results, during loading and unloading of the pile.

Load [kN]	Recorded [mm]	Load [kN]	Recorded [mm]
0	0.00	1470	31.72
20	0.00	1335	31.23
300	0.60	1200	30.97
600	1.72	1055	30.76
900	4.11	645	30.32
1050	5.88	325	30.26
1200	10.54	20	29.14
1335	13.10		
1470	22.29		
1605	32.94		

The data from the table above have been used to analyse with the described method in the paper. As an example, the input parameters of the ‘hyperbolic’ method with free parameters are shown below.

D_s = 500 mm

D_b = 500 mm

L_0 = 1.00 m

L_f = 15.00 m

E_c = 2.5E+07 kN/m^2

M_s = 0.0035 (calculated)

M_b = 0.0626 (calculated)

F_{us} = 1224 kN (calculated)

F_{ub} = 913 kN (calculated)

This will result in the load-transfer graph below:

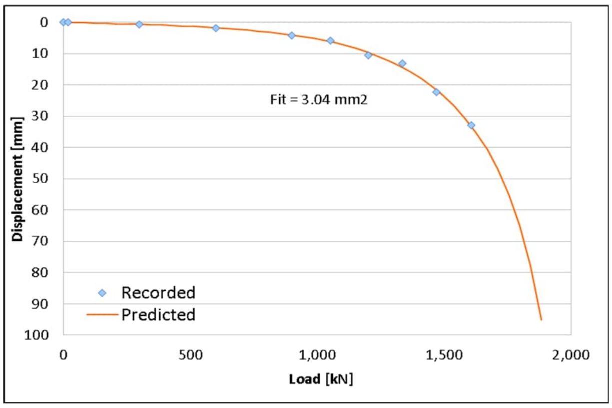

Figure 6 Load-transfer curve ‘hyperbolic’ with free parameters

From the acquired data and calculations, the influence of the base resistance and the shaft friction can be easily plotted for example to the displacement or the total test load.

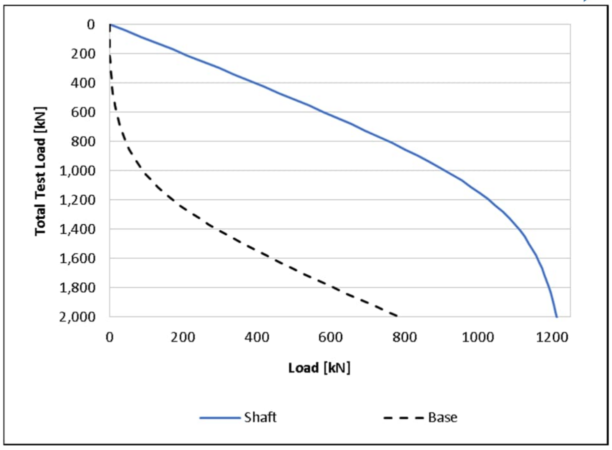

Figure 7 F_s and F_b plotted to total test load

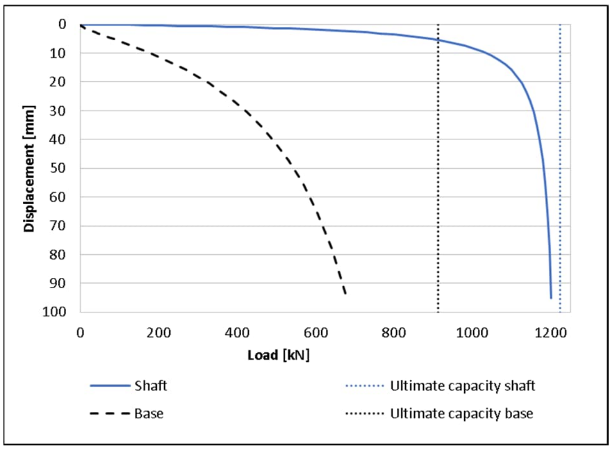

Figure 8 F_s and F_b plotted to total displacement

13 RESULTS OF THE DIFFERENT MODELS

To optimize the models in a spreadsheet, it is very important to select the correct starting values of the parameters. It is the task of an geotechnical engineer to provide these starting values and a provide his ‘engineering judgement’ if the correct optimisation was made.

The calculated parameters are the result of a best fit of the curve to the recorded data. The used model or presented parameters might not be applicable in these kind of soils or for these pile types and are only shown as an example of the different models.

Table 9 Overview results of the different models.

Model	F_{us}	F_{ub}	F_{ut}	Δ	Fit
	[kN]	[kN]	[kN]	[mm]	[mm²]
Verbrugge	959	650	1609	24.1	42.8
Randolph	1012	1239	2252	29.9	39.0
Frank and Zhao	2188	73	2261	27.6	74.5
Frank and Zhao free parameters	1360	1104	2464	32.9	1.6
Hirayama	1207	2179	3386	32.5	6.6
Flemming	1691	187	1878	33.5	8.0
Bohn	100	1738	1838	33.8	10.5
Hyperbolic free parameters	1224	913	2137	33.1	3.0
Hyperbolic – Frank and Zhao	1499	451	1951	33.0	2.9

The graphs and calculations are made with a spreadsheet without the use of any macro’s or scripting.

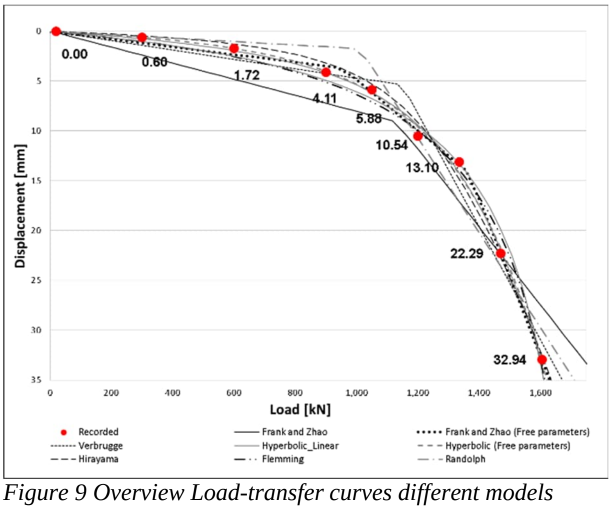

Figure 9 Overview Load-transfer curves different models

When removing the largest and smallest value of the F_{ut} dataset, the average F_{ut} is 2111kN with a standard deviation of 215 kN.

14 CONCLUSION

Load-transfer are a very practical method to convert pile test results to engineering values. The model can be solved by using only spreadsheet tools.

The elastic shortening of the pile in the model can be approximated with a relative simple equation and therefor be part of the integral solution.

Different models can be combined to match the field test results or the local standards. For example, the shaft resistance can be modelled with a hyperbolic approximation and the base can be modelled with a tri-linear model.

From the results of the different models on the same dataset, the conclusion can be made that there is a large spread in results of the models. The modelling of the field test shows an uncertainty of approx. 10% between the outcome of the different models. The hyperbolic model with free parameters and the Frank and Zhao model with free parameters approximate the used field test results most optimal.

REFERENCES

1) C. Bohn; A. Lopes dos Santos; and R. Frank (2016). Development of Axial Pile Load-Transfer Curves Based on Instrumented Load Tests.

2) Fugro LoadTest (2019). CEMSET and CEMSOLVE program manual V003.

3) https://www.e-education.psu.edu/png520/m11_p6.html